647510 **MSC1MatC103x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination

Jan/Feb.-2022 [NEW COURSE]

Topology 1(Core (New))

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (A) Answer any TWO out of three: 10

(i) Prove that arbitrary intersection of closed sets in a topological space is closed.

(ii) Define and check co-countable topology.

(iii) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases which generate topologies $\mathcal{T}$ and $\mathcal{T}'$ on a set X respectively. Prove that $\mathcal{T} \subset \mathcal{T}'$ if and only if $\forall x \in X$ and $x \in B$ in $\mathcal{B}$, $\exists\, B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

Q.1 (B) Answer any ONE out of two: 04

(i) Define simply ordered and co-finite topology.

(ii) Let $X = \{a, b, c\}, \mathcal{T} = \{\emptyset, X, \{a\}, \{b, c\}\}$. Write all $\mathcal{T}$-open sets and $\mathcal{T}$-closed sets.

Q.2 (A) Answer any TWO out of three: 10

(i) Let $X = \{1,2,3,4\}$. Give an example of a topology and subspace topology on X.

(ii) Let X and Y be two topological spaces. Prove that a map $f: X \to Y$ is continuous if and only if the inverse image under f of every closed set in Y is closed in X.

(iii) Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be two topological spaces. If a mapping $f: (X, \mathcal{T}) \to (Y, \mathcal{U})$ is homeomorphism then prove that $f(\bar{A}) = \overline{f(A)}$ for any $A \subset X$.

Q.2 (B) Answer any ONE out of two: 04

(i) Define product topology and basis of a product topology.

(ii) Explain subspace of a topological space with example.

Q.3 (A) Answer any TWO out of three: 10

(i) Let $(X,\mathcal{T})$ be a topological space and $A \subset X$. Prove that $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$

(ii) Let X be a topological space and $A, B \subset X$. Show that

(i) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$ (ii) $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$ and (iii) $\bar{\bar{A}} = A$.

(iii) Prove that a subset A of a topological space $(X,\mathcal{T})$ contains all its limit points if and only if $X - A$ is open.

Q.3 (B) Answer any ONE out of two: 04

(i) Let $(X,\mathcal{T})$ be a topological space and $A \subset X$. Show that $\bar{A}$ is the smallest closed set containing A.

(ii) Let $X = \{a,b,c,d,e\}$, $\mathcal{T} = \{X, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d,e\}\}$ and $A = \{a,b,c\}$. Find all limit points of A.

Q.4 (A) Answer any TWO out of three: 10

(i) Define convergent sequence in a metric space. Show that the limit of a sequence in a metric space is unique, if it exists.

(ii) Let $B_d(x,\varepsilon)$ be an open ball in a topological space with the metric topology and metric d. Let $y \in B_d(x,\varepsilon)$ then show that $\exists\ \delta > 0$ such that $B_d(y,\delta) \subset B_d(x,\varepsilon)$.

(iii) Explain topology of R^n.

Q.4 (B) Answer any ONE out of two: 04

(i) Let $E = [0,1)$ and consider the sequence of functions $f_n(x) = x^n$. Does the sequence converge uniformly on E? Justify your answer.

(ii) Prove that in a metric space (X, d), each open sphere is an open set.

Q.5 (A) Answer any TWO out of three: 10

(i) Prove that two closed subsets of a topological space are separated if and only if they are disjoint.

(ii) Prove that continuous image of a connected space is connected.

(iii) Prove or disprove: Every connected space is locally connected.

Q.5 (B) Answer any ONE out of two: 04

(i) Show that every discrete topological space is locally connected.

(ii) Show that every component of a locally connected space is open.

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